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Mobility edge in the Anderson model on partially disordered random regular graphs

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Recently new mechanisms of ergodicity breaking in the complicated interacting many-body systems have been uncovered. The combination of interaction and strong enough disorder amounts to the emergent many-body localization (MBL) phase with full ergodicity breaking [1–5]. The Anderson model on random regular graph (RRG) serves as the toy model for a identification of MBL phase in the physical space, see [6] for the recent review. The many-body localization in the physical space presumably gets mapped into the one-particle localization in a Hilbert space [7].

We study non-interacting spinless fermions hopping over RRG with connectivity $p = 3$ in a potential disorder described by Hamiltonian

$$H = \sum_{\langle i,j \rangle} \left(c_i^+ c_j + c_i c_j^+ \right) + \sum_{i=1}^{\beta N} \epsilon_i c_i^+ c_i, \tag{1}$$

where the first hopping sum runs over the nearest-neighbor sites of the RRG, the second sum runs over βN nodes with potential disorder. The energies ϵ_i are independent random variables sampled from a uniform distribution on $[-W/2, W/2]$. We consider gaps between adjacent levels, $\delta_i = E_{i+1} - E_i$, where the eigenvalues of a given realization of the Hamiltonian for a given total number of particles, E_i, are listed in ascending order. The dimensionless quantity we have chosen to characterize the correlations between adjacent gaps in the spectrum is the ratio of two consecutive gaps: $r_i = \min(\delta_i, \delta_{i+1})/\max(\delta_i, \delta_{i+1})$. In turn, a direct measure of the (de)localization of the eigenfunctions is obtained by the inverse participation ratio (IPR), $\mathrm{IPR}(i) = \sum_n |\psi_n^{(i)}|^4$, where $\psi_n^{(i)}$ is the i-th eigenstate of the matrix and n is the basis state index.

We analyze the ratio $\langle r \rangle$ and IPR for different parts of the spectrum, dividing the sorting spectrum into $k = 100$ equal parts and average the ratio $\langle r \rangle$ and IPR over each window. The ordinate $\alpha = i/(N-1)$ in Fig. 1 corresponds the normalized level position with $i = 0, 1, \ldots, N-1$, the energy level, the ordinate window respectively is $\Delta\alpha = 1/k$.

The heat maps Fig. 1a, c explicitly demonstrate, that there is the mobility edge λ_m separating sharply the spectrum into two different regimes for RRG with partial disorder in vertices. For $|\lambda| > \lambda_m$ we observe localisation state with the ratio $\langle r \rangle$ close to $\langle r \rangle_P$ and independence of IPR on N, while for central spectrum part with $|\lambda| \le \lambda_m$ the ratio $\langle r \rangle$ and IPP indicate on the delocalized state. Note, that the mobility edge λ_m weakly depends on the disorder W and is observed even for small W. Moreover, we do not observe the phase transition at large W to completely localized phase which is familiar for completely disordered RRG (see Fig. 1b, d, f with the same plots for $\beta = 1.0$).

We consider partially disordered RRG as the toy model of a Hilbert space for some interacting disordered many-body system with the topologically protected subsector. The nodes of RRG free from disorder correspond to topologically protected states in many-body system. To some extend our model probes the effect of disorder on topologically protected states.

It is found that at some density of clean nodes in partially disordered RRG the sharp mobility edge emerges in the spectrum of Anderson model and exists up to arbitrarily large diagonal flat disorder W. We have studied the distribution of the eigenfunctions in RRG and have

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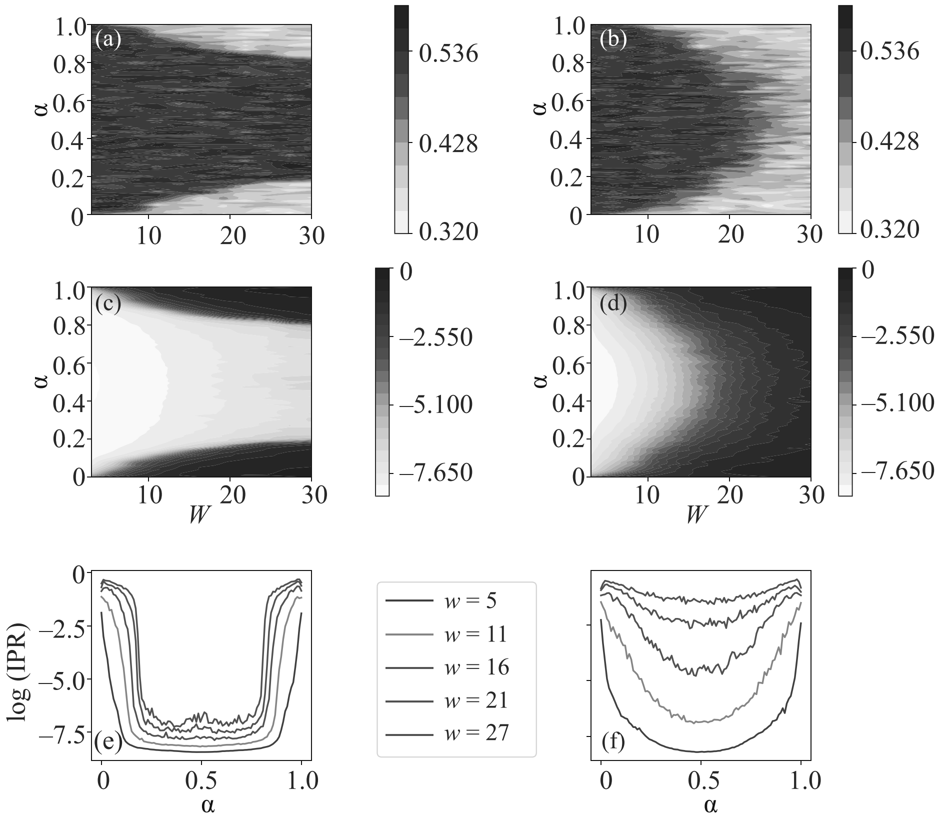

Fig. 1. (Color online) The ratio $\langle r \rangle$ in dependence on the disorder value W and the spectrum part α for $\beta = 0.5$ (a) and $\beta = 1$ (b); the value log(IPR) in dependence on the disorder value W and the spectrum part α for $\beta = 0.5$ (c) and $\beta = 1$ (d); the dependencies of log(IPR) on the spectrum position α for different values of W and $\beta = 0.5$ (e) and $\beta = 1$ (f)

found that localized states are distributed almost solely within the dirty nodes while the delocalized part of the spectrum mainly involves the clean nodes with small disorder dependent contribution of the dirty nodes.

The model certainly oversimplifies the issue, nevertheless it can be considered as the indication that a one-particle mobility edge in the hopping model in the Hilbert space and a mobility edge in the MBL phase could be related. Indeed if the mechanism behind the mobility edge in MBL involves density of highly degenerate zero-modes in the physical space then the density of clean nodes in the Hilbert space is its relevant counterpart.

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