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Topological Josephson junction in transverse magnetic field

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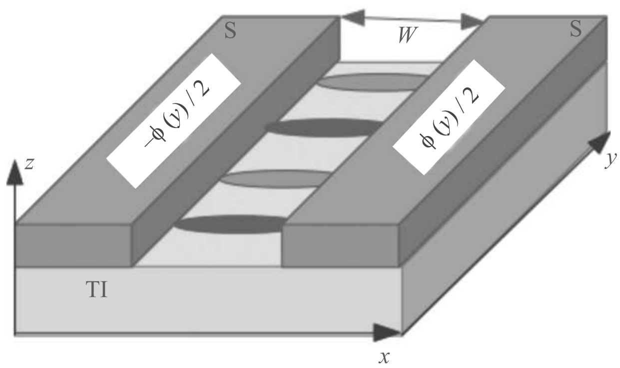
We consider an S-TI-S Josephson junction between two s-wave superconducting (S) electrodes on top of a topological-insulator (TI) material in transverse magnetic field, Fig. 1. Majorana zero modes (MZM's) reside at periodically located nodes of Josephson vortices. We find that hybridization of these modes is prohibited by symmetries of the problem at vanishing chemical potential, which ensures better protection of zero modes and yields methods to control the tunnel coupling between Majorana modes for quantum information processing applications.

Topologically-protected quantum manipulations with Majorana zero modes are extensively studied theoretically and experimentally due to their exotic properties, including exchange statistics, and their possible use in platforms for topological quantum computation [1–3]. In particular, hybrid superconductor-topological insulator structures were discussed. Fu and Kane analyzed a topological Josephson junction between superconductor films on top of a topological insulator [4, 5] and demonstrated the appearance of Majorana edge states. Here we consider a setup where Majorana bound states are point-like structures bound to Josephson vortices in an external magnetic field perpendicular to the surface [6]. Such devices were discussed as a platform for topological quantum computation [7]. We find that the tunnel coupling between the MZM's vanishes at zero chemical potential. This should be taken into account in the design of experiments with MZM's on Josephson vortices and also suggests that coupling and hybridization of various MZM's may be controlled, in particular, via the chemical potential. Note similar observations for a 2D vortex lattice [8].

Due to proximity effect, superconducting correlations are induced in the surface layer of the topological insulator. The states in this layer can be described by the Bogolyubov–de-Gennes (BdG) Hamiltonian $H = \frac{1}{2}\int dxdy\, \Psi^\dagger h \Psi$, where $\Psi = [\psi_\uparrow, \psi_\downarrow, \psi_\downarrow^\dagger, -\psi_\uparrow^\dagger]^T$ and

$$h = v\left(\sigma_x\,[-i\hbar\,\partial_x] + \sigma_y\left[-i\hbar\,\partial_y + \frac{e}{c}A_y(x)\tau_z\right]\right)\tau_z + \\ + \Delta(x,y)\tau_+ + \Delta^*(x,y)\tau_- - \mu\tau_z, \qquad (1)$$

with the Pauli matrices σ and τ referring to the spin and Bogolyubov–Nambu particle-hole space, respectively. For the distribution of the transverse magnetic



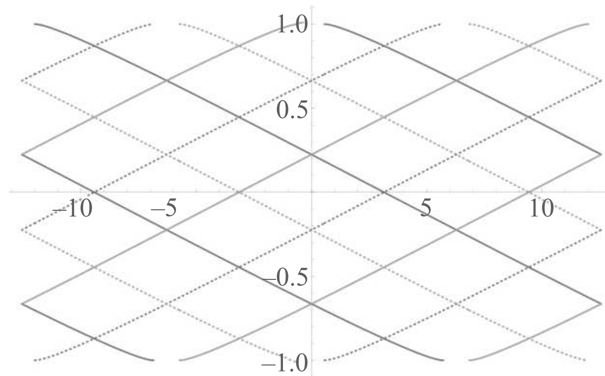


Fig. 1. (Color online) Top: S-TI-S Josephson junction in a transverse magnetic field $\parallel \hat{z}$. Blue and orange spots indicate location of Majorana bound states. Bottom: Eigenenergies $\epsilon_n(\phi)$ of h_0 for $W = 6.0\xi$

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field $\mathbf{H} = H\hat{z}$ around the gap of width W between the superconducting electrodes, we choose the gauge with $\Delta = \Delta_0 e^{\pm i\phi/2}$ in the leads with the phase $\phi(y) = (2\lambda_L + W)\, H_0 \frac{2\pi}{\Phi_0}\, y$, Fig. 1. We assume that the corresponding magnetic length $l_B = [(2\lambda_L + W)\, H_0/\Phi_0]^{-1}$ exceeds the relevant coherence length $\xi = \hbar v/\Delta_0$.

Properties of the solutions are to large extent determined by symmetries of the Hamiltonian. They include the particle-hole symmetry $C = \sigma_y \tau_y K$, which inverts energies: $C^{-1} h C = -h$. Further, for $\mu = 0$, the case of our special interest below, there is a (quasi) time-reversal symmetry $T = \sigma_x \tau_x K$, with $T^{-1} h T = h$, and $T^2 = 1$. Their product defines the chiral symmetry $S = \sigma_z \tau_z$. Finally, an extra symmetry operator,

$$F = \sigma_x \tau_x I_x\,, \tag{2}$$

is defined via the x inversion I_x.

We split the Hamiltonian into

$$h_0(y) = -i\hbar v\, \sigma_x\, \tau_z \partial_x + [\Delta(x,y)\tau_+ + \text{h.c.}] - \mu\tau_z \tag{3}$$

and

$$h_1 = v\, \sigma_y \left[-i\hbar\partial_y + \frac{e}{c} A_y(x)\tau_z\right] \tau_z\,, \tag{4}$$

and first diagonalize h_0 for each value of y (or ϕ):

$$\hat{h}_0(y)|\nu\rangle_y = \epsilon_\nu(y)|\nu\rangle_y \tag{5}$$

or equivalently, $\hat{h}_0(\phi)|\nu\rangle_\phi = \epsilon_\nu(\phi)|\nu\rangle_\phi$. Later, we take h_1 into account, which glues these 1D states into 2D wave functions.

The resulting eigenstates are classified by the eigenvalues of the symmetry operators F (blue, $F = 1$, and orange, $F = -1$, color in the figures) and σ_x, an extra symmetry of h_0. The respective spectrum ϵ_ν, cf. Fig. 1, depends on ϕ (or y), and has zero modes [4] at $\phi = \pi + 2\pi n$:

$$[1, \sigma_x, \sigma_x F, F]^T \cdot e^{-\int_0^{|x|} |\Delta(x')| dx'} \tag{6}$$

with $F = (-1)^n$ (note another gauge used in [4]). We mark an eigenstate $\epsilon_\nu(y)$ of h_0 with $\nu = (n, \pm)$, where $\pm = -\sigma_x$ is the sign of its slope.

We further work in the eigenbasis (5) of h_0, in contrast to [4]. First, we account for the kinetic part h_{kin} of h_1 and then treat the remaining $\tilde{h}_1$ perturbatively. Since h_{kin} couples only the levels within each charge conjugate pair $n\pm$, the BdG equation factorizes into 2×2 equations with

$$h_{\text{eff}} = -iv\rho_y \partial_y + \epsilon_n(y)\rho_z\,. \tag{7}$$

Here ρ are a new set of Pauli matrices in the basis $n\pm$.

The energy ϵ_n vanishes at coordinates $y = y_n$ with phase $\phi(y_n) = \pi + 2\pi n$. Around this point we linearize $\epsilon_n(y) = \alpha\,(y - y_n)$ and obtain an exactly solvable [6, 7]

$$h_{\text{eff}} = -iv\rho_y \partial_y + \alpha\,(y - y_n)\rho_z \tag{8}$$

with $\alpha \propto \partial_y \phi$. This equation has a zero mode near y_n and a set of "Landau levels". In the regime we consider, $l_B \gg \xi$, many Landau levels fit below the gap Δ_0.

Taking into account $\tilde{h}_1$ may tunnel-couple zero modes at different nodes $\phi = \pi + 2\pi n$, pushing them away from zero energy. We showed that this is not the case, and at $\mu = 0$ the modes remain at zero energy, forming a flat band, cf. [8] for a 2D vortex lattice.

Indeed, from Eq. (8) one observes that the zero mode at each node, $|n+\rangle + S|n-\rangle$, is chiral with the same chirality $S = \sigma_z\tau_z = -\text{sign}\,\alpha$ for all nodes. We showed that adding $\tilde{h}_1$ does not alter this property. While the zero modes remain decoupled, other levels near a certain node can be perturbed due to coupling to another node.

Our result is in agreement with the general classification of zero modes [9], which implies that the number of MZM's is topologically protected and given by the total phase drop accumulated around the defects within the Josephson junction.

Coupling between the MZM's in this setting was suggested as a method to braid them [7], which would allow for topologically protected quantum operations. While the coupling vanishes at $\mu = 0$, a finite coupling may be achieved for $\mu \neq 0$, which also permits controlling its strength.

Furthermore, finite coupling can be realized if two MZM's have different chiralities. Since the chirality depends on the sign of the magnetic field ($\text{sign}\,\alpha$ in Eq. (8)), this can be effected with a nonuniform magnetic field distribution: for instance, with a "domain wall" of $H(y) = H_0 \text{sign}\, y$ or an oscillatory $H(y)$. The corresponding structures would allow for controlled quantum operations with Majorana modes.

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