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On series and integral representations of some NRQCD master integrals

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At present we have a lot of techniques for calculating multiloop Feynman diagrams, see [1–3] for recent reviews. A particular useful method is the solution of some system of differential equations [4–10]. In many cases the results for Feynman diagrams can be written in terms of multiple polylogarithms (MPLs) [11–13], which is well studied class of functions at the moment. In this particular case the corresponding system of differential equations can be reduced to the so-called ϵ-form [9, 10, 14]. When it is not possible we are required to introduce both new more general classes of functions and new solution techniques.

In the present short note, we use an example a set of two-loop master integrals arising in the process of matching of QCD to NRQCD. We are considering master integrals for a family of Feynman integrals studied previously in [15, 16]. The latter is defined as

$$J^{mM}_{b_1b_2b_3a_1a_2} = \int \frac{d^dkd^dl}{\pi^2}\frac{1}{D_1^{b_1}D_2^{b_2}D_3^{b_3}D_4^{a_1}D_5^{a_2}}, \quad (1)$$

where the propagators are $D_1 = (k+q_1)^2 - m^2$, $D_2 = (k-q_2)^2 - m^2$, $D_3 = k^2 - m^2$, $D_4 = (l + \frac{q_1-q_2}{2})^2 - M^2$, $D_5 = (l-k)^2 - M^2$ and the kinematics is given by $q_1^2 = q_2^2 = 0$ and $q_1 \cdot q_2 = 2m^2$. A graphical representation of this family of integrals can be found in Fig. 1, where we defined $p = \frac{1}{2}(q_1 + q_1)$. Using integration by parts (IBP) relations [17, 18] all integrals in this family can be reduced to the set of 9 master integrals. The latter can be chosen as

$$(J_1, \ldots, J_9)^\top = \Big\{ J^{mM}_{00011}, J^{mM}_{00101}, J^{mM}_{00111}, J^{mM}_{00211}, J^{mM}_{00121}, J^{mM}_{01011}, J^{mM}_{02011}, J^{mM}_{01111}, J^{mM}_{11101} \Big\}^\top.$$

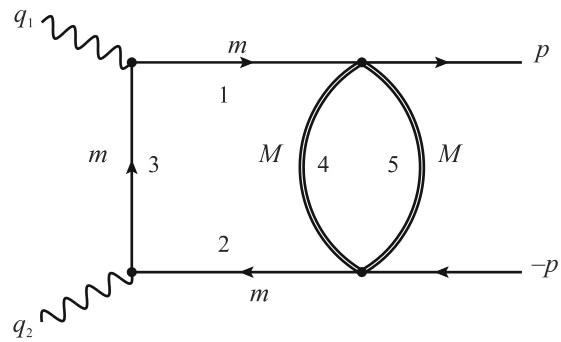

Fig. 1. Graphical representation for the family of integrals in Eq. (1)

In the first part of the work we following [27] consider analytical Frobenius[2)] solutions of our master integrals for general values of space-time dimension. It turns out, that if we look for solutions of the system of differential equations in the form of Frobenius power series in the square of mass ratio m^2/M^2, then the recurrence relations for the series coefficients can be reduced to first-order difference equations. The solution of the latter does not cause any difficulties. The final result can then be rewritten in terms of generalized hypergeometric functions ${}_3F_2$, ${}_4F_3$ and ${}_5F_4$. Next, we discuss the use of Feynman parameter trick to reduce the problem of evaluation of two-loop master integrals to effective one-loop problem. The latter can then be solved with the use of differential equations method with respect to mentioned Feynman parameter [15, 16, 25, 26]. Note, that similar Feynman parameter trick was used before in [28–31] under the name of effective mass approach [4, 6, 32]. In the last part we show how the exact Frobenius results in terms of hypergeometric ${}_pF_q$- functions

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2) For previous applications of Frobenius method in the context of Feynman diagrams see for example [16, 19–24].

can be transformed into corresponding integral representations exact in space-time dimension.

The obtained results agree with those obtained previously when either exact results exists or up to available expansion order in ϵ. The presented techniques for obtaining Frobenius power series and integral representations are both simple and powerful enough with a great potential for their extension to other problems.

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