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Phase-coherent thermoelectricity in superconducting hybrids (Mini-review)

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For decades thermoelectricity in superconductors was and remains one of the most intriguing topics of modern condensed matter physics [1]. The essence of the so-called thermoelectric effect in metals is illustrated by a simple relation $\mathbf{j} = \alpha \nabla T$ indicating that electric current $\mathbf{j}$ can be generated by exposing the system to a thermal gradient ∇T. Usually the latter effect remains quite small since contributions from electron-like and hole-like excitations are of the opposite sign and almost cancel each other. As a result, the thermoelectric coefficient α turns out to be proportional to a small ratio between temperature T and the Fermi energy ε_F, i.e. $\alpha \propto T/\varepsilon_F$.

Quite unexpectedly, already first experiments with bimetallic superconducting rings [2–4] revealed the thermoelectric signal which magnitude exceeded theoretical estimates by *several orders of magnitude*. Later on large thermoelectric signals were also observed in multi-terminal hybrid superconducting-normal-superconducting (SNS) structures [5–8] frequently called Andreev interferometers. Furthermore, the thermopower detected in these experiments was found to be periodic in the superconducting phase difference across the corresponding SNS junction. The latter observation (i) indicates that the thermoelectric signal in superconductors can be *phase coherent* and (ii) calls for establishing a clear relation between thermoelectric, Josephson and Aharonov–Bohm effects in systems under consideration. Both issues (i) and (ii) – along with an experimentally observed large magnitude of the thermoelectric effect – constitute the key subjects of our present review.

It follows from the above arguments that large thermoelectric effects can be expected provided electron-hole symmetry is violated in some way. In this case the contributions from electron-like and hole-like excitations would not cancel each other anymore and, hence, the thermoelectric coefficient α would not be restricted by a small parameter T/ε_F and can become large.

Electron-hole asymmetry in superconducting hybrid structures can be realized by a number of physical mechanisms. As a simple example, let us consider a superconducting ring pierced by external magnetic flux Φ_x and interrupted by a normal metal as it is schematically shown in Fig. 1. Quasiparticles propagating from

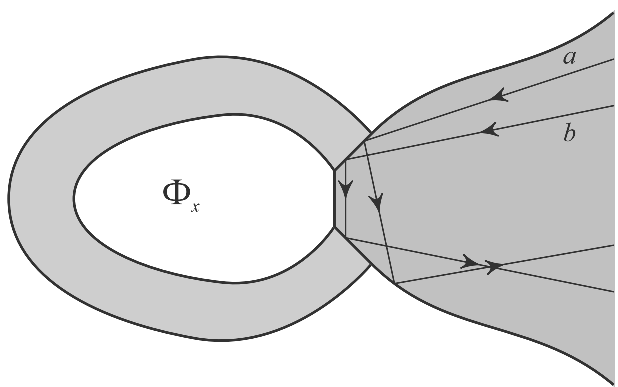

Fig. 1. (Color online) A simple setup illustrating electron-hole symmetry breaking due to Andreev reflections (trajectory b). The setup consists of a superconducting ring pierced by external magnetic flux Φ_x and attached to a piece of a normal metal

a normal metal towards a superconducting ring eventually hit either one NS interface (trajectory a) or both of them (trajectory b). In either case an incoming electron with subgap energy may be Andreev-reflected back as a hole. For quasiparticles propagating along the trajectory a the probability for this reflection process equals identically to that for the inverse process, i.e. $\mathcal{R}_a^{e-h} = \mathcal{R}_a^{h-e}$. At the same time, it is straightforward to demonstrate [9] that for electrons following the trajectory b the above equation does not hold anymore, i.e. $\mathcal{R}_b^{e-h} \neq \mathcal{R}_b^{h-e}$ provided $\Phi_x \neq \Phi_0 n/2$ (where Φ_0 is the superconducting flux quantum) implying that scattering on two NS interfaces generates *electron-hole symmetry violation* in our

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hybrid structure. This electron-hole asymmetry yields a large thermoelectric effect in the system under consideration [9].

Likewise, one can demonstrate that spin-sensitive electron scattering in superconductors doped with point-like magnetic impurities [10] or containing spin-active interfaces [11–13] may generate electron-hole symmetry breaking which in turn yields dramatic enhancement of the thermoelectric effect. Electron-hole asymmetry accompanied by large scale thermoelectric effects are also predicted to occur in superconductor-ferromagnet hybrids with the density of states spin split by the exchange and/or Zeeman fields [14, 15]. These theoretical predictions were verified in experiments with superconductor-ferromagnet tunnel junctions in high magnetic fields [16] where large thermoelectric currents were observed.

In the remaining part of our review we focus our attention on the phase-coherent nature of thermoelectric effects observed in multi-terminal superconducting-normal hybrid structures. Such phase coherence manifests itself in a periodic dependence of the thermopower $\mathcal{S}$ on the applied magnetic flux Φ_x indicating their close relation to Josephson and Aharonov–Bohm effects [17]. We demonstrate that coherent oscillations of the thermopower are controlled by a number of contributions originating from these two effects as well as from electron-hole asymmetry [18]. The relative weight of these contributions depends on the relation between temperature, voltage bias, and an effective Thouless energy of the setup. We particularly emphasize the role of the system topology that may have a dramatic impact on the behavior of $\mathcal{S}(\Phi_x)$ in a qualitative agreement with experimental observations [5–8].

We also analyze a nontrivial interplay between non-equilibrium effects and long-range quantum coherence in superconducting hybrid nanostructures exposed to a temperature gradient. In particular, we demonstrate that dc Josephson current in multi-terminal hybrid structures can be efficiently tuned and stimulated by applying a temperature gradient to such structures [19–21]. At temperatures T exceeding the Thouless energy of our device both the supercurrent and the thermoelectric voltage signal may be strongly enhanced due to non-equilibrium low-energy quasiparticles propagating across the system without any significant phase relaxation. As a result, the supercurrent decays slowly (algebraically rather than exponentially) with increasing T and can be further enhanced by a proper choice of the circuit topology. At large values of the temperature gradient, the non-equilibrium contribution to the supercurrent may become as large as the equilibrium one at low T.

In addition, we predict a nontrivial current-phase relation and a variety of transitions between 0- and π-junction states controlled by the temperature gradient as well as by the system geometry [19–21].

We hope that theoretical results and predictions discussed in our review not only shed light on some previously unresolved issues but also could help to put forward numerous applications of thermoelectric effects ranging from thermometry and refrigeration [22] to phase- coherent caloritronics [23].

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